Journal of Engineering Physics and Thermophysics, Vol. 72, No. 3, 1999

SPECIAL FEATURES OF THE PROCESSES OF HEAT AND MASS TRANSFER UNDER A SHELF GLACIER

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UDC 551.51

The reaction of a shelf glacier to possible changes in the global temperature of the ocean is studied. A two-dimensional model of the thermohaline circulation of water under a shelf glacier that is determined by heat and mass transfer on the boundary of the ocean and the glacier is considered. The velocity of the front of the phase transition "glacier–water" is evaluated, and the effect of different boundary conditions on the velocity is studied. Melting/building-up of ice on the lower boundary of a shelf glacier is an integral response to a change in the temperature of the water, and it can be a characteristic of the global change in the temperature of the ocean.

Introduction. Increasing interest in evaluation estimation of the dynamics of the global temperature of the World Ocean is observed as a result of studies of the changes in the climate of the planet [1]. Indirect estimates of the expected increase in the temperature amount to several hundredths of a degree per year [2] although this value is not indisputable. The temperature of the ocean is characterized by high variability associated with processes in both the ocean and the atmosphere. Therefore, changes in the temperature are difficult to measure in the temperature mode of the World Ocean as a whole. To achieve this, one should average short-term fluctuations to obtain representative data on long-term temperature variations. By virtue of this, the conditions on the lower boundary of shelf glaciers appear to be unique, since the effects of ablation/accumulation (i.e., melting/building-up) of ice on them are an integral response to the change in the temperature of the ocean. Moreover, shelf glaciers isolate the part of the ocean under them from short-term fluctuations of the temperature of the atmosphere.

Monitoring of the magnitude of ablation/accumulation can be done by the following procedure: having bored a hole through the entire thickness of the glacier one places a number of pickups on the lower boundary of the shelf that allow one to observe the change in the glacier boundary. This method, as an idea, was suggested earlier [3, 4]. Numerical evaluations for its use show that the hole freezes completely in 16 days, and the temperature of the glacier surrounding the hole returns to the initial distribution in approximately the same time [5]. The accuracy of this measurement may be very high compared to the only currently available method of determination of the temperature of the ocean – the so-called "acoustic thermometer." This method averages local variations of the temperature over large regions of the ocean up to 10,000 km. Its action is based on measurement of the variation of the velocity of sound, which increases by 4.6 m/sec with increase in the temperature of the water by 1 ^{o}C [1]. Shelf glaciers are widespread in the Antarctic and the Arctic [6]. About 44% of the Antarctic coast is occupied by shelf glaciers [7]. Therefore, the idea of development of a monitoring network in the polar regions for observation of changes in the temperature of the World Ocean is very attractive.

In the water under a shelf one can distinguish virtually horizontal layers that differ in salinity and temperature, with lower layers being more saline and warmer than upper ones. The temperature of the layers adjacent to the shelf is lower than the freezing point. Floating-up of warmer water occurs in the region adjacent to the boundary of the glacier and the ground, where the thickness of the layer of water is minimum. Warm deep layers of water move along the bottom of the cavity, and then they rise to the lower boundary of the shelf and, while cooling, shift to the free surface of the glacier.

To explain the processes occurring in a shelf cavity, Jenkins [8] suggested a one-dimensional model where the water under the shelf glacier was treated as a system of two layers: a lower layer that has a high salinity and

Moscow State Institute of Engineering Physics, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 3, pp. 545-554, May-June, 1999. Original article submitted June 11, 1998.

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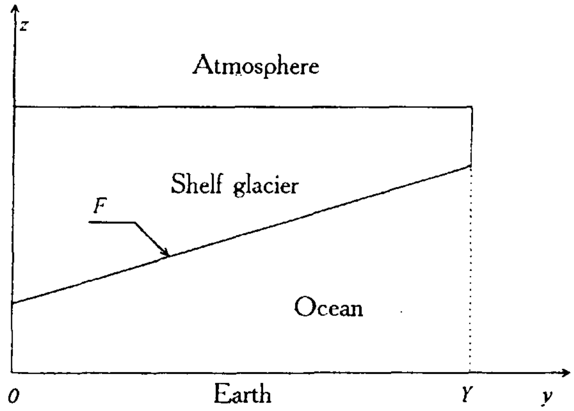

Fig. 1. Schematic representation of the problem.

temperature and an upper one thhat is adjacent to the shelf and is a mixture of ice and water with lower salinity and temperature. Heat and mass transfer on the boundary of the two layers takes place at a velocity proportional to the velocity of the water. The velocity, flow, temperature, and salinity averaged over the depth are used to describe the motion. The maximum value of melting obtained by this model is equal to 6 m/year, the mean value on the boundary of the shelf is 0.6 m/year, and the mean velocity of the ice–water mixture is 8 cm/sec. Within the framework of the one-dimensional model the direction of the phase transition is determined only by the depth.

Helmer and Olbers [9] used a two-dimensional model to describe the circulation of water and thermohaline processes. At the initial instant of time the water under a shelf glacier is at rest. Motion is caused by a horizontal density gradient associated with both the phase transition on the glacier boundary and the slope of the shelf to the ground. Water moving virtually parallel to the bottom rises along the boundary of the shelf and changes its direction of motion. At the same time at the free end of the shelf additional circulation arises that is caused by the fact that the water is in a supercooled state and the density gradient has the opposite sign. The maximum value of melting of the lower end of the shelf glacier is reached at the boundary of the glacier and the ground and amounts to 1.5 m/year; building-up begins at distances of more than 450 km, the maximum value of 0.1 m/year corresponds to 490 km, then this value decreases, and at the end of the shelf melting takes place again.

MacAyeal [10] studied the effect of tidal phenomena on the processes occurring under a shelf. Noticeable mixing due to tides is observed at the boundary of the ice and the ground because here the thickness of the water is minimum, and in the remaining region directed flow is predominant. At the same time, melting of the glacier at the boundary with the ground that is caused by the tide can be the main reason for convective motion. Estimates of the value of melting made by different techniques show that it does not exceed 0.5 m/year.

In the present paper the motion of seawater in the shelf cavity caused by the processes of melting/building-up of the lower boundary of the glacier is studied. The velocity of the phase transition is evaluated. The effect of a global change in the temperature of the ocean on the dynamics of the lower boundary of a shelf glacier is investigated.

Formulation of the Problem. We consider a system involving a shelf glacier that is contiguous to the ground on one side and has a free surface on the other and seawater under the glacier (Fig. 1). The motion of incompressible seawater can be described in the Boussinesq approximation. Then the equation of motion has the form

$$u_t + uu_x + vu_y + wu_z - fv = - p_x + A_{\rm h} (u_{xx} + u_{yy}) + A_{\rm V} u_{zz} , \tag{1}$$

$$v_t + uv_x + vv_y + wv_z - fu = - p_y + A_{\rm h} (v_{xx} + v_{yy}) + A_{\rm V} v_{zz} , \tag{2}$$

$$p = - \rho g z + p_0 , \tag{3}$$

and the continuity equation can be written as

$$u_x + v_y + w_z = 0\,, \tag{4}$$

where u, v, w are the components of the velocity v; $A_h = 10^3$ m^2/sec, $A_V = 10^{-3}$ m^2/sec. Since $u \ll v$ (characteristic values are $u = 0.01$ m/sec, $v = 0.1$ m/sec), it is assumed that thermohaline circulation has predominantly a two-dimensional character. Changing over to a two-dimensional geometry, we neglect derivatives with respect to the coordinate x and the Coriolis force. Equations (2)-(4) take the form

$$v_t + vv_y + wv_z = -p_y + A_h v_{yy} + A_V v_{zz}\,, \tag{2'}$$

$$p_y = -\rho_y g z\,, \tag{3'}$$

$$v_y + w_z = 0\,, \tag{4'}$$

where $\rho = 1033$ kg/m^3. We introduce the stream function Ψ: $v = \Psi_z$, $w = -\Psi_y$, and then after differentiation with respect to z Eq. (2$'$) is written as

$$\Psi_{zzt} + (v\Psi_{zz})_y + (w\Psi_{zz})_z = g\rho_y + A_h\Psi_{zzyy} + A_V\Psi_{zzzz}\,. \tag{5}$$

Thermohaline processes arise due to a horizontal density gradient (the quantity $g\rho_y$). The latter is caused by the phase transition at the ice–water boundary and the slope of the shelf to the horizontal. The density ρ is determined by the equation of state

$$\rho = \rho_0\,[1 - \beta_T(T - T_0) + \beta_S(S - S_0) + \gamma(z - z_0)]\,, \tag{6}$$

where z is the depth, m; $\beta_T = 7.35\cdot10^{-5}$ °C^{-1}, $\beta_S = 8.02\cdot10^{-7}$ (‰)$^{-1}$ [11]; $\gamma = 5.86\cdot10^{-3}$ m^{-1} [12]; $T_0 = 0$ °C; $S_0 = 35$‰; $z_0 = 0$.

The temperature and the salinity are described by the equations of thermal conductivity and diffusion:

$$X_t + (vX)_y + (wX)_z = K_h X_{yy} + K_V X_{zz}\,, \tag{7}$$

where X means T or S; $K_h = 10$ m^2/sec, $K_V = 10^{-4}$ m^2/sec are the horizontal and vertical coefficients of turbulent diffusion [9].

Tidal phenomena and the motion of the water caused by wind on the open continental shelf play a much lesser role than thermohaline processes, and therefore they are disregarded in this model [10]. Moreover, we neglect the flow of the glacier ice due to creep processes.

Boundary and Initial Conditions. Since the melting or building-up on the lower boundary of the glacier is much smaller than its thickness, it is assumed that the thickness of the shelf does not change with time and the change on the lower boundary of the glacier is compensated by accumulation and ablation of snow on the surface.

On the surface of the ground and on the lower boundary of the glacier the velocity of water flow vanishes, which corresponds to the condition of adhesion of a viscous fluid: $v(y, 0, t) = 0$, $w(y, 0, t) = 0$, $v(y, F, t) = 0$, $w(y, F, t) = 0$, where F is the boundary of the shelf glacier and the ocean (Fig. 1).

Incoming and outgoing flows of water at the free end of the shelf glacier have only a horizontal component of the velocity, since the water flow in the neighboring region of the ocean is parallel to the free surface: $w(Y, z, t) = 0$, Y is the free boundary of the shelf glacier (Fig. 1).

The temperature at the ice–water boundary is equal to the temperature of the phase transition and is a function of the salinity and the pressure:

$$T^B = aS^B + b - cp\,, \tag{8}$$

where $a = -0.057$ °C, $b = 0.0939$ °C, $c = 7.64\cdot10^{-2}$ MPa [9].

The boundary values of the temperature and the salinity are determined from conservation of the heat flux and the flux of salinity. The total heat flux Q_T passing through the ice–water boundary is

$$Q_T = Q_T^{\rm B} + Q_T^{\rm i} \tag{9}$$

or

$$Q_T = -K_{\rm w} \left.\frac{\partial T}{\partial z}\right|_{\rm B}, \tag{10}$$

where $K_{\rm w} = 414$ W/(m·°C) is the turbulent thermal conductivity of seawater; $\partial T/\partial z|_{\rm B}$ is the heat flux from water to ice; $Q_T^{\rm B}$ is the heat flux caused by melting or building-up of ice:

$$Q_T^{\rm B} = \rho_{\rm i} L \dot{h}, \tag{11}$$

where $\rho_{\rm i} = 917$ kg/m^3, $L = 334$ kJ/kg; $Q_T^{\rm i}$ is the flow determined by heat transfer through the ice:

$$Q_T^{\rm i} = -\rho_{\rm i}\, c_{p{\rm i}} \frac{T^{\rm is} - T^{\rm B}}{H}, \tag{12}$$

where $k = 1.54 \cdot 10^{-6}$ m^2/sec; $c_{p{\rm i}} = 2$ kJ/(kg·°C) is the heat capacity of ice at a temperature of −20 °C; $T^{\rm B}$ is the temperature of the lower end of the shelf. It is assumed that the temperature in the glacier is distributed linearly, and since the changes in the temperature at the lower surface occur in a region whose dimensions are much smaller than the thickness of the glacier, the heat flux through the ice does not change with time.

The total flux of salinity Q_S on the ice–water boundary without regard for molecular diffusion of salt through the ice has the form

$$Q_S = -D_S \left.\frac{\partial S}{\partial z}\right|_{\rm B}, \tag{13}$$

where $D_S = 0.1$ m^2/sec is the coefficient of turbulent diffusion; $\partial S/\partial z|_{\rm B}$ is the flux of salinity from water to ice. By virtue of the law of conservation it should be equal to the flux of salinity caused by the phase transition:

$$Q_S^{\rm B} = S^{\rm B} \dot{h}. \tag{14}$$

At the bottom of the ocean and at the free end of the shelf glacier the following boundary conditions are adopted:

case A: the relations

$$\frac{\partial T^{\rm w}}{\partial y} = 0, \quad \frac{\partial S^{\rm w}}{\partial y} = 0 \tag{15}$$

are fulfilled on the boundary where water flows out of the cavity ($u > 0$), and where $u < 0$, the temperature and the salinity have the initial values;

case B: over the entire depth of the boundary of the cavity under the end of the shelf conditions (15) are met.

Here condition (A) corresponds to the fact that water having the temperature and salinity of the ocean enters the cavity and water with the temperature and salinity of the cavity flows out; condition (B) describes a thermally insulated cavity and is considered as some limiting case in order to study the role of heat transfer of water in the cavity with the surrounding ocean. The geothermal flow at the bottom of the ocean is taken to be 0.05 W/m^2.

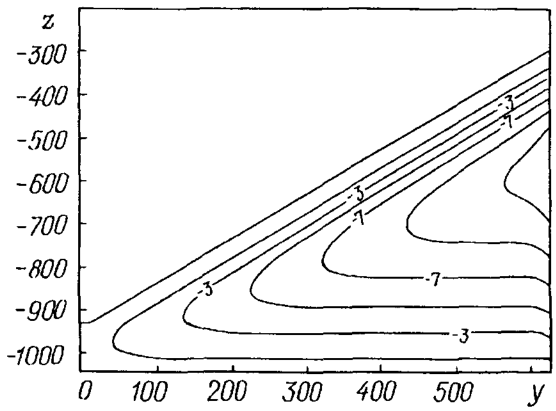

Fig. 2. Streamlines Ψ under a shelf glacier at the instant of time of 5 years for the conditions of standard calculation. The figures at the curves are the values of Ψ. y, km; z, m.

The value of the initial vertical gradients of salinity and temperature exerts a considerable effect on the character of the water flow in the cavity.

Three different versions of the values of the vertical gradients of temperature and salinity were taken as the initial quantities.

The conditions for standard calculation correspond to the values of salinity and temperature obtained for the Filchner Ice Shelf [9].

For condition (I) the gradient of salinity is the same as in standard calculation, and the temperature gradient is equal to the minimum at which free convection of a liquid column should arise (see the Appendix).

Condition (II) is formulated for the case where a postcritical value of the parameter responsible for the appearance of convection is realized (see the Appendix).

Thus, three different conditions are used:

$$\frac{\partial S}{\partial z} = -0.00031\ \text{‰/m}, \quad \frac{\partial T}{\partial z} = -0.00039\ ^{\circ}\text{C/m} \quad \text{(standard calculation)};$$

$$\frac{\partial S}{\partial z} = -0.00031\ \text{‰/m}, \quad \frac{\partial T}{\partial z} = -0.0013\ ^{\circ}\text{C/m}; \tag{I}$$

$$\frac{\partial S}{\partial z} = -2.82\cdot 10^{-5}\ \text{‰/m}, \quad \frac{\partial T}{\partial z} = -0.00175\ ^{\circ}\text{C/m}. \tag{II}$$

At the initial instant of time the system is at rest, i.e., $v(t = 0) = 0$, $w(t = 0) = 0$.

The tangent of the angle of slope of the lower boundary of the shelf glacier with respect to the ground is 10^{-3}, the length along the horizontal is 620 km, the bottom lies at a depth of 1080 m, and the thickness of the glacier at the free surface is 200 m.

Equations (8)-(14) were used to determine the boundary values of temperature and salinity and to find the velocity of the phase transition h.

Method of Solution. Problem (5)-(13) was solved numerically. The method of variable directions was used in combination with the method of iterations. Calculation was made by a first-order implicit scheme, and the velocity in convective terms was taken from the preceding iteration. Equation (5) was solved in two stages: first, the value of Ψ_{zz} was determined, and then the stream function was found. The steps in the spatial coordinates and time were constant: $\Delta y = 11.5$ km, $\Delta z = 12$ m, $\Delta t = 12$ h.

Discussion of the Results. The calculation was made up to the moment of establishment of a quasistationary state of the system. On the boundary between the ice and the water a phase transition occurs, which is the reason for the motion of seawater, which begins at the ice—water boundary, but then deep warm layers of water begin to raise upward, thus causing displacement of water in the entire region. The stream lines (Fig. 2) reflect a circulatory

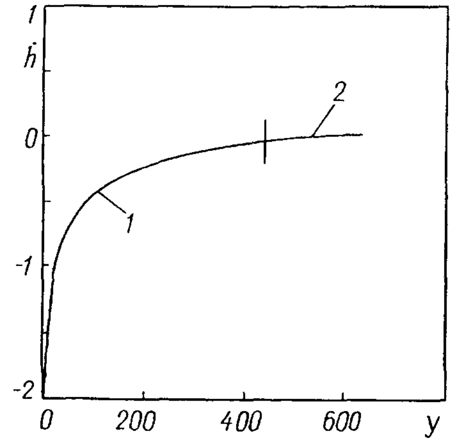

Fig. 3. Velocity of the phase transition as a function of distance for the conditions of standard calculation: 1) melting; 2) building-up. h, m/year.

motion of water. Deep warm layers of water move along the bottom of the cavity, and then they float to the boundary of the shelf. Water formed in ice melting mixes with seawater, rises along the boundary of the shelf glacier, and moves toward its free end. At the free boundary of the shelf, as is seen in Fig. 2, the velocity of the water changes sign at a depth of ~550 m. From the bottom to this depth the water flows into the shelf cavity, and above it the water flows out. The mean velocity of the outgoing flow is higher than that of the incoming.

In a large portion of the region considered (up to 425 km) on the lower surface of the shelf glacier, melting of ice takes place (Fig. 3). The highest ablation occurs at the sea bottom, and the velocity of the phase transition is of the order of 2 m/year. With increase in the distance from the bottom this quantity decreases, and at the free end of the shelf building-up of about 8 cm/year is observed. In the region of building-up the water is in a supercooled state, and its temperature is below the temperature of the phase transition at the corresponding depth. The equilibrium temperature and salinity (on the ice–water interface) decrease with depth. The region of supercooled water corresponds to the so-called region of existence of intrawater ice, which is formed in the form of individual plates from the supercooled solution. The dynamics of intrawater ice requires additional study.

The established temperature and salinity differ from the initial distributions of these quantities in a layer with a thickness of about 50 m that is parallel to the lower boundary of the shelf.

A series of calculations with various boundary conditions was made for the temperature at the end of the shelf glacier. The temperature was varied by $\Delta T = \pm 0.01$ and 0.001 °C/year. The boundary of the transition from melting to building-up is at a distance of 460 km at $\Delta T = 0.01$ °C/year and 390 km at $\Delta T = -0.01$ °C/year. The change in the velocity of the phase transition as compared to standard calculation ($\Delta T = 0$) for $\Delta T = 0.01$ °C/year is 2.5 cm/year on the average, and for $\Delta T = -0.01$ °C/year it is 1.5 cm/year. For $\Delta T = \pm 0.001$ °C/year the difference does not exceed 0.5 cm/year.

In the region adjacent to the end of the shelf the water is in a supercooled state. With increase in the temperature of the water the region decreases. For $\Delta T = -0.01$ °C/year its vertical size is ~100 m, for $\Delta T = 0$ it is already equal to 70 m, and for $\Delta T = 0.01$ °C/year 50 m. The nonequilibrium thermodynamic state is compensated for due to formation of ice crystals. In the given model this process corresponds to building-up on the lower boundary of the shelf glacier. The results presented correspond to the initial gradients of standard calculation and the boundary conditions (A).

The value of the initial vertical gradients of salinity and temperature affects substantially the character of water motion in the cavity.

We consider case (B). At the beginning of the process ($t \sim 1$ month) the streamlines for the initial gradients (I) and (II) behave almost the same, forming two large regions of motion: one is closed and lies in the depth of the cavity (up to 400 km), and the flow there has a circulatory character; the other lies closer to the free end of the shelf, here the streamlines are not closed, and the water, while moving along the bottom, rises to the boundary of the shelf and flows along it to the free surface (Fig. 4a). It can be assumed that the different types of convection

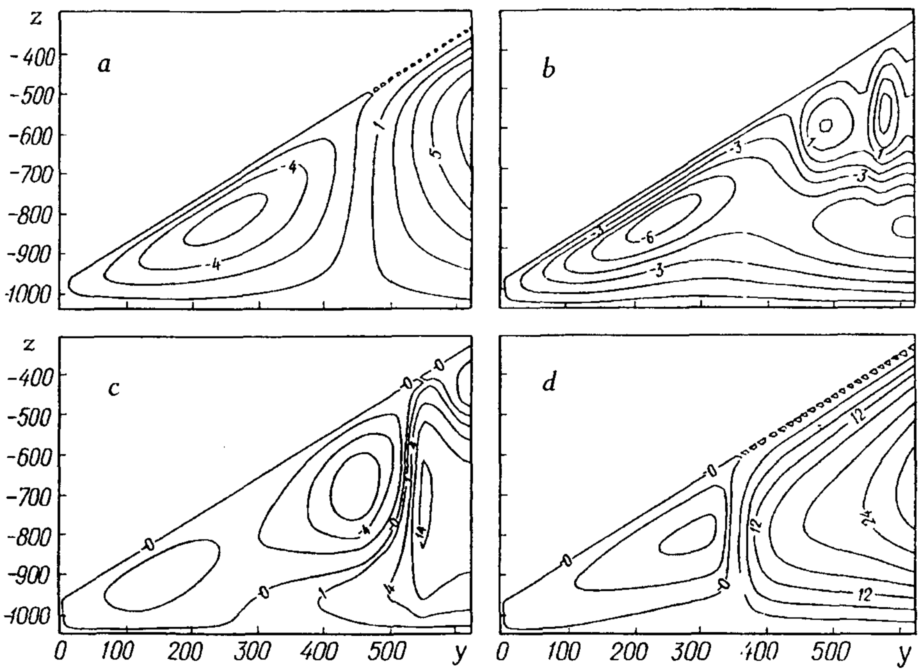

Fig. 4. Streamlines at the instant of time of: a) 1 month for boundary conditions (B); b) 3 months for boundary conditions (B) and (I); c) 6 months for boundary conditions (B) and (II); d) 1 year for boundary conditions (A) and (II). The figures at the curves are the values of Ψ.

observed in laboratory studies of the instability of the mashy layer in freezing of a salt solution [13] are realized. Convective cells are formed at the lower boundary between the glacier and the ocean (at distances > 400 km). Here a region of supercooled liquid exists where the gradients of temperature and salinity are high and change direction several times, thus determining the convective motion of the liquid. In this case the effective Rayleigh number is much higher than the critical value. Convective motion is observed only in the region of the mashy layer. In the depth of the cavity the gradient of temperature and salinity has a constant sign. Here convection has a more global character, and the process is observed in a large region (its boundaries correspond to the region where the gradients of temperature and salinity have a constant sign). However, after 2 months differences are observed. For case (I), already by 3 months streamlines that pass from the free boundary through the entire shelf cavity and again reach the free ocean are formed (Fig. 4b). Here, closed regions where circulation of water takes place already exist. Since closed cells lie in a region closer to the free boundary but lower in depth, it can be assumed that they are caused by temperature equalization equalization over the depth. After 6 months these regions disappear, and the water, while moving along the bottom, rises to the lower boundary of the shelf and flows to the free ocean; the streamlines behave similarly to the case of standard calculation. For case (II), a closed region remains (Fig. 4c) whose size increases with time; several circulation cells exist inside this region. They do not disappear because the initial gradient of temperature is too large and no equalization of temperature due to heat conduction takes place. At distances > 450 km at the boundary of the glacier a region of supercooled liquid exists, and its presence leads to a sharp increase in the velocity of the phase transition at the boundary between the glacier and water. This is associated with the fact that the cavity is thermally insulated and the nonequilibrium state is compensated for only due to the phase transition at the boundary between the shelf glacier and the ocean.

We consider the other case (condition (A)). In this case for the two types of conditions (I) and (II) a closed region exists in the depth of the shelf cavity; its dimensions decrease with time. For case (II) the character of the motion at the lower boundary of the shelf changes. The number of convective cells increases and the region of their existence expands (Fig. 4d). This is caused by the fact that the region where the temperature is equalized over the depth is determined by the sign of the velocity. Since the coefficient of thermal diffusivity in the horizontal direction is higher than that in the vertical direction, heat is transferred predominantly in this direction, and the boundary conditions at the free end of the shelf exert an effect at rather large distances from the free end of the shelf.

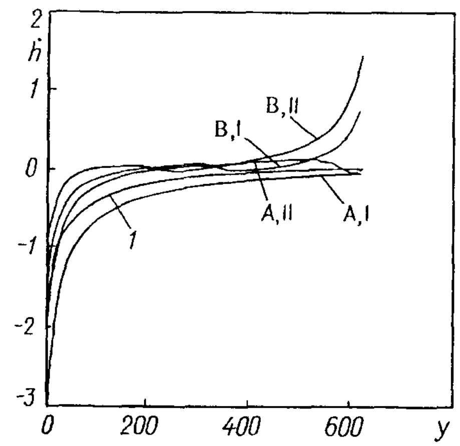

Fig. 5. Velocity of the phase transition as a function of distance for various initial and boundary conditions [1) standard calculation].

The character of the velocity of the phase transition is approximately the same for all cases. In the depth of the cavity maximum melting, whose value decreases toward the free surface of the shelf, is observed. Farther away, building-up is observed, for case (A) it is more intense, and the boundary of its appearance lies deeper in the cavity (Fig. 5).

The existence of a closed region inside the cavity results in the fact that a change in the temperature of the water at the free surface of the shelf becomes less noticeable in both case (I) and case (II). For case (I) the mean deviation of the velocity of the phase transition with a change in the temperature of the water of the surrounding ocean by $\Delta T = \pm 0.01$ °C/year amounts to about 9 mm in the region of unclosed streamlines and about 2 mm in the closed region. In case (II) this quantity decreases to 7 and 0.7 mm, respectively. The results presented refer to case (A). For condition (B) a change in the temperature of the ocean at the free surface of the glacier in no way affects the velocity of the phase transition, since here the cavity is insulated for heat and mass transfer.

CONCLUSIONS

1. The main reason for the appearance of water motion in the cavity is the processes occurring on the boundary between the shelf glacier and the seawater. Depending on the initial vertical gradients of the temperature and salinity of the water various modes of water motion in the cavity are possible; this affects the behavior of the velocity of the phase transition and correspondingly the possibility of monitoring the temperature of the ocean.

2. A model study of the motion of the water under a shelf glacier showed that various modes of formation of convective cells similar to those noted in laboratory experiments of [13] are observed (depending on the initial and boundary conditions).

3. Calculations indicate that for the gradients of temperature and salinity at which convective instability is absent it is possible to monitor the global temperature of the ocean with annual-mean variations in the temperature of about 0.01 °C/year. The value of the deviation of the velocity of the phase transition for $\Delta T = 0.01$ °C/year is 2.5 cm/year on the average, and for $\Delta T = -0.01$ °C/year it is 1.5 cm/year.

4. For gradients of the temperature and salinity of the ocean corresponding to the mode of convection, observation of a change in the temperature of the ocean is possible only with a rather long time of change in the position of the boundary of the shelf glacier, because in this case the velocity of the change in the phase interface is small.

The authors are grateful to the International Science-Engineering Center (project MNTTs V-23) and to the Civil Research and Development Foundation (grant CRDF RG2-144) for partial support of this work.

Appendix. The motion of liquid in the cavity can be caused not only by the phase transition at the ice–seawater interface, but also by convective instability of the liquid. It is known that for a liquid in the field of gravity free convection is possible [14]. We determine the condition for its appearance allowing for the salinity of the seawater.

The equation of state of seawater (6) can be written in the form

$$\rho = \rho_0 \, [1 - \beta_T (T - 0^{\circ}\mathrm{C}) + \beta_S (S - 35\,‰)] \, .$$

We represent the temperature, salinity, and density as

$$T = T_0 + T' \, , \quad S = S_0 + S' \, , \quad \rho = \rho_0 + \rho' \, , \quad p = p_0 + p' \, ,$$

where X_0 is the mean value; X' is the deviation from it.

The pressure of the liquid is determined as

$$p_0 = - \rho_0 \, g z + \mathrm{const} \, .$$

Then the Navier–Stokes equations and the equations of continuity, thermal conductivity, and diffusion for the given case are written in the form

$$\frac{\partial \mathrm{v}}{\partial t} + (\mathrm{v}\nabla)\,\mathrm{v} = - \frac{\nabla p'}{\rho_0} + \nu \Delta \mathrm{v} + \mathrm{g}\,(\beta_S (S' - 35) - \beta_T T') \, , \quad \mathrm{div}\, \mathrm{v} = 0 \, ,$$

$$\frac{\partial T'}{\partial t} + \mathrm{v}\nabla T' = K_V \Delta T' \, , \quad \frac{\partial S'}{\partial t} + \mathrm{v}\nabla S' = K_V \Delta S' \, .$$

We represent the deviations of temperature, salinity, and pressure in the form of two components:

$$T' = T_0' + \tau \, , \quad S' = S_0' + s \, , \quad p' = p_0' + \rho w \, ,$$

where τ, s, w are the disturbances caused by the motion of the liquid. The quantities T_0', S_0', p_0' satisfy stationary equations of thermal conductivity and diffusion:

$$\Delta T_0' = \frac{d^2 T}{dz^2} = 0 \, , \quad \Delta S_0' = \frac{d^2 S}{dz^2} = 0 \, , \quad \frac{dp_0'}{dz} = \rho_0 \, g \, (\beta_T T' - \beta_S (S' - 35)) \, .$$

Then $T_0' = A(H - z) + T_b$, $S_0' = B(H - z) + S_b$, where T_b and S_b are the temperature and salinity of the water at the bottom of the cavity. We take into account only the first order of smallness of the quantities τ, s, w, v and assume that they are time-dependent $\sim \exp(-i\omega t)$. Then

$$- i\omega \mathrm{v} = - \nabla w + \nu \Delta \mathrm{v} + \mathrm{g}\, \beta_S s - \mathrm{g}\, \beta_T \tau \, , \tag{A.1}$$

$$- i\omega \tau - A v_z = K_V \Delta \tau \, , \tag{A.2}$$

$$- i\omega s - B v_z = K_V \Delta s \, , \tag{A.3}$$

$$\mathrm{div}\, \mathrm{v} = 0 \, , \tag{A.4}$$

where A, B are the vertical gradients of temperature and salinity, respectively.

The boundary conditions are $v|_\Gamma = 0$, $\tau|_\Gamma = 0$, $s|_\Gamma = 0$, where Γ is the boundary of the region considered.

The given system of equations is distinguished by the additional term $\mathrm{g}\beta_S s$ on the right-hand side of Eq. (A.1) and the additional equation (A.3) from a similar system obtained for fresh water [14]. Using the condition of absence of convection Im $(\omega) = 0$ [14], we obtain the equation

$$\Delta^3 \tau - \frac{\partial^2 \tau}{\partial z^2} (\Re_T - \Re_S) = 0 ,$$

where $\Re_S = g\beta_S B l^4 / K_V A_V$, $\Re_T = g\beta_T A l^4 / K_V A_V$, i.e., for the case of salt water $\Re_T - \Re_S$ is a parameter of the system. Having denoted $\Re = \Re_T - \Re_S$, we can use the results of [14]. In the given case $\Re$ plays the role of an effective Rayleigh number.

The condition for the appearance of convection is

$$\Re / l > C_* ,$$

where $C_* = 1708$. Using the values of the parameters for standard calculation, we obtain for $\beta_T = 7.35 \cdot 10^{-5}$ $^oC^{-1}$, $A = 3.9 \cdot 10^{-4}$ $^oC/m$; $\beta_S = 8.02 \cdot 10^{-4}$ $(‰)^{-1}$, $B = 3.1 \cdot 10^{-4}$ ‰/m, $100 < l < 1000$ m, $K_V = 10^{-4}$, $A_V = 10^{-3}$ m^2/sec, $g = 9.8$ m/sec^2

$$\Re / l \le 500 ,$$

i.e., convection is absent for the standard case.

NOTATION

y, horizontal coordinate, km; z, vertical coordinate, m; t, time, sec; u, v, w, longitudinal, horizontal, and vertical components of the velocity, m/sec; f, constant associated with the Coriolis force, sec^{-1}; p, pressure, Pa; g, acceleration of gravity, m/sec^2; ρ, density, kg/m^3; T, temperature, ^oC; S, salinity, ‰; L, latent heat of melting, J/kg; h, velocity of the phase transition, m/year; c_{pi}, heat capacity of the ice, $J/(kg \cdot {}^oC)$; H, thickness of the shelf, m; k, coefficient of thermal diffusion of the ice, m^2/sec; ρ_i, ρ_0, density of the ice and the water, kg/m^3; T^B, S^B, boundary values of temperature and salinity; ^oC; T^w, S^w, temperature and salinity of the seawater, ^oC; T^{is}, temperature of the surface of the ice, ^oC; $\partial T/\partial z$, vertical gradient of temperature, $^oC/m$; $\partial S/\partial z$, vertical gradient of salinity, ‰/m; $\partial T/\partial z|_B$, $\partial S/\partial z|_B$, gradients of temperature and salinity at the ice–water interface; l, thickness of the water layer, m; Ψ, stream function, m^2/sec; A_h, A_V, horizontal and vertical components of the viscosity. Superscripts and subscripts: w, water; i, ice; T, temperature; S, salinity; is, surface of the glacier; B, boundary between the glacier and the ocean; b, bottom; h, horizontal component; V, vertical component.

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